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WEBER'S LAW AND ANTAGONISTIC SALT ACTION

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1. In a series of papers beginning in 1900 I have shown that: (*a*) It is necessary for the normal functions of living organs and organisms that the ratio of the concentration of antagonistic ions or salts C_I/C_{II} of the surrounding solution be kept within certain limits; if the value of this quotient becomes either too high or too low, life phenomena become abnormal and finally impossible.[1] (*b*) The salts to be considered as antagonistic in this sense are in the first place those of univalent and bivalent metals and that therefore the most important critical quotient will generally be C_{Na+K}/C_{Mg+Ca}. (*c*) There is also an antagonism between the salts of bivalent ions such as Sr and Ca, and Mg and Ca.[2] In the following discussion the antagonism between Na and K on the one hand, and Mg and Ca on the other will be solely considered.

I further suggested that if the value of C_I/C_{II}, e.g., C_{Na+K}/C_{Mg+Ca} had once reached the lower limit required for the normal process of life phenomena the alternate reversible replacement of one of the two groups of ions by their antagonists in combinations with certain colloids (proteins or fatty acids) of the cells determined the alteration between activity and rest of the organs or the organisms.

This idea was carried further by Lasareff, who showed that from these data not only Nernst's law of electrical stimulation could be derived[3] but also the phenomena of vision in weak light in which the effect depends solely on the bleaching of visual purple by the light.[4]

Life phenomena as a rule take place in a medium whose composition and concentration undergoes little or no variation, such as sea water or blood serum, and the majority of organisms cannot stand any wide variation from this fixed standard. For such organisms only one value

for C_{Na+K} can be considered and hence only the lower and upper limit for C_{Mg+Ca} can be determined for this value. A few organisms can stand wide variations of osmotic pressure and hence in their case the value C_{Na+K} can undergo wide variations. The question arises: How do the minimal and maximal values of C_{Mg+Ca} change when C_{Na+K} varies? It is this question which will be discussed in this paper.

2. The animals used in these experiments were the newly hatched larvae of a barnacle (*Balanus iberneus*). These larvae upon hatching are positively heliotropic and form a dense cluster on the light side of the aquarium, so that they can be collected in a pipette with comparatively little sea water. They are able to live not only in normal sea water but also in sea water which is diluted with eight times its volume of distilled water, as well as in sea water whose concentration is raised almost 50%.

When such larvae are put into a mixture of NaCl, KCl and $CaCl_2$ of the concentration and proportion of sea water, the majority will not swim but fall to the bottom of the dish. If, however, we increase the amount of $CaCl_2$ in the solution the larvae instead of falling to the bottom will swim, as they would in normal sea water, and will collect at the side of the dish nearest to or most remote from the light. Instead of adding more $CaCl_2$ we may add $MgCl_2$ or $SrCl_2$.

3. If the bivalent cations contained in the solution are of one kind only, namely either $CaCl_2$ or $MgCl_2$ or $SrCl_2$, and if they are in sufficient concentration to allow the animals to swim, the latter will do so as a rule only for about five minutes. If we wish the animals to swim permanently we must add a mixture of two bivalent cations, e.g., Ca and Mg.

In the following experiments a mixture of $CaCl_2$ and $MgCl_2$ was always used to supply the bivalent cations. This mixture contained the two cations in that ratio in which they occur in the sea water, namely 1.5 molecules of $CaCl_2$ to 11.6 molecules of $MgCl_2$. The concentration of the mixture used was $\frac{3}{8}$ grammolecular.

4. When we put the animals into a mixture of 50 cc. NaCl + KCl (no matter what concentration) they will fall to the bottom and are unable to make sufficiently vigorous swimming motions, although they may live in such a solution for a day or longer. Various concentrations of the mixture of NaCl + KCl (in the proportion in which these salts exist in the sea water) were prepared and it was ascertained what was the minimum amount of $CaCl_2 + MgCl_2$ necessary to induce all (or practically all) the animals to make normal swimming movements and to collect in a dense cluster at the window side (or the opposite side) at the surface of the dish.

The absolute value of Ca + Mg required was not always identical for the same concentration of NaCl + KCl in different experiments; this was probably due to differences in temperature or in illumination or in the condition of the larvae. It was, therefore, necessary to compare only experiments made simultaneously with the same material. As a rule two sets of experiments were made simultaneously for two different concentrations of NaCl + KCl; the two concentrations of NaCl + KCl were usually chosen in the ratio of 1:2 or 1:4.

Table I gives some of the results. The figures for $CaCl + MgCl_2$ indicate the number of cc. $\frac{3}{8}$ m $CaCl_2 + MgCl_2$ required to just allow

TABLE I

MINIMAL AMOUNT OF $CaCl_2 + MgCl_2$ REQUIRED TO ALLOW THE LARVAE TO SWIM IN 50 CC. (NaCl + KCl) OF THE FOLLOWING CONCENTRATIONS

NUMBER OF EXPERIMENT	CONCENTRATION NaCl + KCl	CC. OF 3/8 M $CaCl_2 + MgCl_2$ REQUIRED	VALUE OF $\frac{C_{Na+K}}{C_{Mg+Ca}}$
I	m/16	0.3	27.8
	m/8	0.4 to 0.5	37.0
II	m/8	0.5	33.3
	m/4	0.9 to 1.0	35.1
III	3/16 m	0.7	35.7
	3/8 m	1.3	38.5
IV	m/8	0.5	33.3
	m/2	1.8 to 1.9	36.0
V	m/4	0.8 to 0.9	39.2
	m/2	1.6 to 1.7	40.3
VI	5/16 m	0.9	46.3
	5/8 m	1.7	49.0
VII	3/16 m	0.6	41.7
	6/8 m	2.4	41.7

all the animals to swim and to collect in a dense heliotropic cluster; a diminution of this quantity by 0.1 cc. resulted, as a rule, in a diffuse and incomplete collection while with a further diminution by 0.1 cc. only a small percentage of larvae rose to the suface. The endpoint or threshold according to our criterion was therefore usually sufficiently sharp to allow different unbiased observers to select the same dish as representing the threshold in a series.

It was necessary to wait four hours or still better a day, before the final

decision was made, as in the very weak as well as in the strong solutions the larvae fall first to the bottom and will not rise until after some hours.

Other experiments gave similar results.

If we select for comparison the experiments with the moderate concentrations (between m/8 and m/2 NaCl + KCl) *we find that the value of* C_{Na+K}/C_{Mg+Ca} *varies little around 35.0 or in other words that the concentration of CaCl + MgCl required increases in direct proportion with the concentration of NaCl + KCl;* as far as the degree of accuracy of these experiments permits it to be stated. In concentrations above m/2 NaCl the animals will usually not all rise or if they do they soon fall to the ground again and this makes the determinations of the endpoint difficult. The value of $MgCl_2 + CaCl_2$ to be added to m/16 NaCl is higher than the theory demands and therefore the law does not hold strictly in this case. *The law of direct proportionality found for moderate concentrations is therefore Weber's law.* As is well known, Weber's law

TABLE II

MAXIMAL AMOUNT OF $CaCl_2 + MgCl_2$ WHICH ALLOWS THE LARVAE TO SWIM IN 50 cc. (NaCl + KCl) OF THE FOLLOWING CONCENTRATIONS

CONCENTRATION NaCl + KCl	MAXIMAL AMOUNT OF $CaCl_2 + MgCl_2$	VALUE OF $\frac{C_{Na+K}}{C_{Mg+Ca}}$
50 cc. m/16 NaCl + KCl	> 5 cc. 3/8 m $CaCl_2 + MgCl_2$	1.67
50 cc. m/8 NaCl + KCl	> 8 cc. 3/8 m $CaCl_2 + MgCl_2$	2.08
50 cc. m/4 NaCl + KCl	>14 cc. 3/8 m $CaCl_2 + MgCl_2$	2.38
50 cc. m/2 NaCl + KCl	>25 cc. 3/8 m $CaCl_2 + MgCl_2$	2.67

is not strictly correct for very low or very high intensities and the deviation is in the same sense as in our experiments, namely as if the weakest concentration tried were stronger than it actually is.

5. According to the writer's theory the swimming motions should also cease when the value of C_{Na+K}/C_{Mg+Ca} falls below a certain limit. It is more difficult to determine this endpoint than the other one, since with an increasing concentration of Ca + Mg the percentage of the larvae able to swim diminishes very gradually.

When the larvae are put into solutions with an excess of $CaCl_2 + MgCl_2$ they all swim at first but in a few minutes they fall to the bottom though they will live for a long time. The power of rising to the surface and forming a permanent dense heliotropic cluster no longer existed when more than the following quantities of 3/8 m $CaCl_2 + MgCl_2$ were added to 50 cc. of NaCl + KCl (Table II). The final observations were usually taken after two days.

Considering the difficulty in getting a sharp endpoint, these values are in good agreement with Weber's law of proportionality. It is, therefore, apparent that the upper limit of the value C_{Na+K}/C_{Mg+Ca}, which just annihilates the power of free swimming motions, remains also approximately constant, namely about 2, if the value of C_{Na+K} varies. Attention should be called to the fact that this value seems to rise slowly with the concentration of NaCl + KCl.

6. It is occasionally stated that the bivalent cations are the stimulating ions. This can be disproved if we prepare solutions of $MgCl_2$ + $CaCl_2$ in m/2 cane sugar or m/2 glycerine. It is impossible to induce the larvae to swim in such solutions. Neither can we induce them to swim if we add NaCl + KCl to a m/2 cane sugar or glycerine solution. This is in harmony with the law that the normal activity is only possible if the value of the quotient C_{Na+K}/C_{Mg+Ca} keeps within certain limits, but contradicts the idea that Ca or Mg stimulate the animals into activity.

7. It has been stated that in a m/2 solution of NaCl + KCl + $CaCl_2$, in the normal proportions, none or few of the larvae can swim, but that on addition of either more $CaCl_2$ or $MgCl_2$ or both the larvae will swim. We can bring about the same effect if we add instead of $MgCl_2$ or more $CaCl_2$ simply some alkali, e.g., 0.8 cc. N/100 NaOH or 0.5 cc. m/10 $NaHCO_3$ to 50 cc. of the solution of NaCl + KCl + $CaCl_2$. The larvae will after some time rise to the surface and gather permanently in a cluster although no $MgCl_2$ is present. This fact brings these phenomena into a close parallel with the phenomena of fertilization. I have shown that the eggs of sea urchins and other marine animals cannot be fertilized in a mixture of NaCl + KCl although the sperm remains active for a long time in such a solution; that the addition of $CaCl_2$ allows the fertilization of a few eggs, but that the addition of either $MgCl_2$ or NaOH (or $NaHCO_3$) may allow all the eggs to be fertilized.[5] This is mentioned to indicate that the phenomena discussed in this paper are of a wide biological application.

Summary. I had formerly shown that the normal functions of an organism are only possible if the value C_{Na+K}/C_{Mg+Ca} remains with certain limits A and B. In this paper this value has been investigated in an animal which stands wide variations of C_{Na+K} and it was found that the values A and B remain approximately constant if C_{Na+K} changes. This fact is the expression of the Weber-Fechner law. Since this law underlies many phenomena of stimulation it appears possible that changes in the concentration of antagonistic ions

or salts are the means by which these stimulations are brought about, as suggested in my ion-protein theory and by the investigations of Lasareff.

[1] Loeb, *The dynamics of living matter*, New York, 1906. Physiologische Tonenwirkung, *Oppenheimer's Handbuch*, Vol. 2, Jena, 1909.

[2] Loeb, *Amer. J. Physiol.*, **3**, 434 (1899); **6**, 411 (1902); and *J. Biol. Chem.*, **1**, 427 (1906).

[3] Lasareff, *Arch. ges. Physiol., Bonn*, **135**, 196 (1910).

[4] Lasareff, *Arch. ges. Physiol., Bonn*, **154**, 459 (1913).

[5] Loeb, *Science*, **40**, 316 (1914); *Amer. Nat., Boston*, **49**, 257 (1915).

THE POLARIZED FLUORESCENCE OF AMMONIUM URANYL CHLORIDE

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The remarkable fluorescence spectrum of ammonium uranyl chloride ($U O_2 Cl_2 2NH_4Cl + 2H_2O$), which has been described in a recent paper read before the American Physical Society, consists of several equidistant groups of bands. Each group contains five nearly equidistant bands, *b*, *c*, *d*, *e*, and *a* (fig. 1) and the spacing repeats itself with such precision in successive groups that the homologous bands, b_1, b_2, b_3,—c_1, c_2, c_3 (see fig. 2) form series having a common and constant frequency interval.

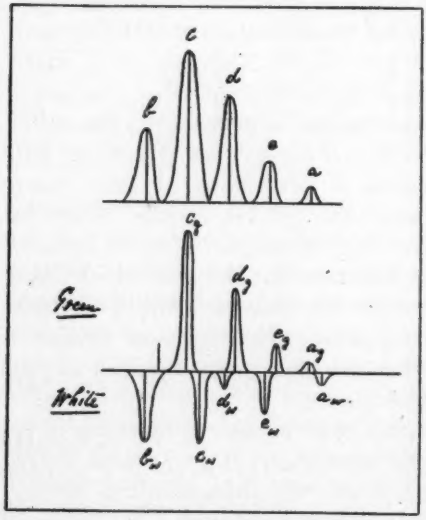

FIG. 1

Observations at the temperature of liquid air show that these bands are really doublets, unresolved at +20° but separated at low temperatures; a dim companion of the band as observed at +20° increasing greatly in brightness as the temperature falls while the dominant member becomes relatively feeble or in some instances disappears altogether. At low temperatures, as is usual with the uranyl compounds, all the bands are very narrow so that overlapping components which are entirely indistinguishable at the tempera-

ture of the room appear as separate bands. The relative brightness of the components, and also the effect of temperature upon them, varies greatly with different bands.

Ammonium uranyl chloride crystallizes in triclinic plates, some of which show a marked pleochroism. According to Mr. D. T. Wilber, to whom we are indebted for the preparation of this and the other uranyl compounds mentioned in this paper:

The strong pleochroism is observed in crystals tabular parallel to the face c(001) the crystals being set to correspond with the isomorphous potassium uranyl chloride as described in Groth's *Chemische Krystallographie*, volume 1, p. 594. The transmitted light is polarized with one component vibrating nearly parallel to the edge between the c(001) face and the b(010) face. This is less absorbed, the transmitted light appearing nearly white in

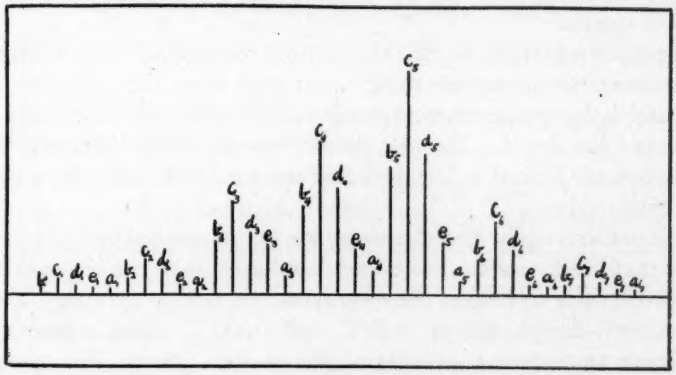

FIG. 2

plates 1 mm. or less in thickness. The component at right angles to this is strongly absorbed in the blue and violet and appears deep greenish yellow even in much thinner layers.

We shall designate these directions of polarization simply as *white* and *green*.

When the violet rays from an electric carbon arc are transmitted through the slit of a spectroscope and the fluorescence and absorption bands are observed through a Nicol prism, marked changes are found to occur as the Nicol is turned with its plane polarization parallel successively to the green and white planes of the crystal.[1]

The two components of the fluorescence and absorption may be conveniently observed simultaneously by means of the following arrangement of apparatus which is that employed in our determinations.

In the collimator of a spectroscope a doubly refracting rhomb was so mounted as to give vertically displaced images of the slit, and the length of the slit was adjusted so as to render these images contiguous but not overlapping. Two spectra one above the other were thus produced polarized vertically and horizontally and when the crystal to be studied was mounted before the slit and carefully adjusted by rotation until its planes of polarization were likewise vertical and horizontal these spectra gave the fluorescence and absorption for the *green* and *white* components respectively. A suitable light filter of deep blue glass which was opaque to the entire region occupied by the fluorescence so that the bands appeared upon a black background was placed in the path of the incident light. It transmitted light of the wave-lengths necessary to excitation, however, and this included the region of the absorption spectrum, so that fluorescence bands and absorption bands were simultaneously visible in both spectra.

Complete separation of the two polarized components, even when the adjustments are accurately made, occurs only when the crystal is perfect and many specimens were tested and discarded before one suitable for study was found. The best results were obtained with small thin crystals newly formed and immediately protected from the air by a layer of Canada balsam.

Comparisons of the fluorescence bands by the method just described show that in the various groups each band consists of two components; thus confirming in general the conclusions reached by the study of the unpolarized fluorescence at $+20°$C. and $-185°$. These components moreover are polarized at right angles to each other. The stronger fluorescence, as might perhaps be expected, is polarized in the *green* direction, corresponding with the greater absorption.

The location and roughly approximate relative intensity of the fluorescence of each group is indicated in figure 1, in which for a single typical group the bands of the unpolarized spectrum are shown and, below, the *green* and *white* components. From this figure it will be seen that all except the band b appear as oppositely polarized doublets. This band in the *green* spectrum is absent or too dim for detection. We know however that at $-185°$ a component at the position b' takes the place of b hence it is probably only a question of insufficient brightness. The component d_w is so dim as to be seen only with great difficulty and the same is true of a_w.

For purposes of measurement photographs were made of the spectra of this compound and also of the fluorescence and absorption of crystals of potassium uranyl chloride, caesium uranyl chloride, and rubidium

uranyl chloride prepared for us by Mr. Wilber. In photographing absorption bands a nitrogen filled tungsten lamp was substituted for the arc and a light blue filter was sometimes used in place of the deep blue screen already described. Owing to the very great range of intensity in the fluorescence bands and the great difference in the transmission. within the region of absorption for the *white* and *green* directions respectively it was necessary to make many exposures of varying duration in order to obtain a complete record of the spectra.

When the transmitted light is polarized and parallel to one of the planes of vibration of the crystal, only one absorption spectrum is observed and the completeness of extinction of the other affords a delicate test for the adjustment of the apparatus and for the homogeneity of the specimen. It is however significant that the two polarized fluorescence spectra are always present, provided the emission is that leaving the

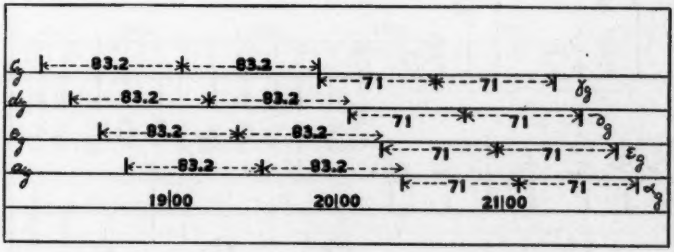

FIG. 3

crystal in the direction described in an earlier paragraph, and these spectra are the same whether the exciting light is polarized in the *white* or the *green* direction or whether unpolarized light is used for excitation. The exciting light, moreover, may enter the crystal in any direction without affecting the character of the fluorescence spectra.[2]

This is in accord with the general principle established by the study of numerous cases of fluorescence,[3] i.e., that a fluorescence band is independent, as regards its location and character, of the nature of the excitation.

It follows moreover that a beam of plane polarized light is capable of exciting not only the polarized fluorescence which has its vibration in the same plane *but likewise that component of the fluorescence which is polarized at right angles to the exciting light.* This would seem to remove the fluoresence of these crystals and presumably the very similar fluorescence of the other uranyl compounds from the class designated by Wood as resonance spectra.

The absorption spectrum of ammonium uranyl chloride, observed under the conditions described above, is no less remarkable than the spectrum of the fluorescent light.

It extends from just about 0.5 μ into the ultraviolet and consists, like the fluorescence spectrum, of several series of equidistant bands having a common frequency interval.

This interval, expressed arbitrarily in the form $10^3/\lambda$ where μ is the unit of wave length, is approximately 71.0 whereas the interval for the fluorescence is 83.2.

Our earlier observations, without regard to polarization had led to the recognition of four series designated as α, β, γ, δ, and three of these appeared to consist of doublets.

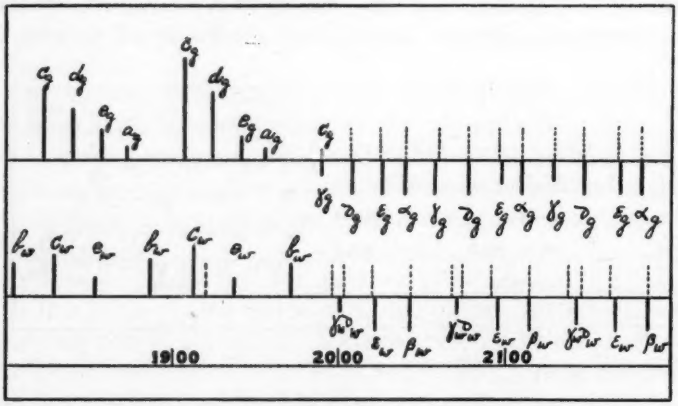

FIG. 4

The relation between absorption and fluorescence at $+20°$ is briefly as follows:

In the region lying between 0.5075μ and 0.4900μ which is occupied by the fluorescence group of shortest wave length, and which we have termed the reversing region, there is an absorption band for each fluorescence band and coincident as to position. The reversing region is the most unsatisfactory for the study of fluorescence on account of this coincidence of fluorescence and absorption. It is however possible by the use of suitable screens, to almost entirely suppress fluorescence and to locate the absorption bands of this region. Towards the red all the fluorescent bands form series with a common constant interval of 83.2. Towards the violet all the absorption bands belong to similar series starting from these reversible bands with a common constant interval of 71.0. (See fig. 3.)

If these remarkable relations are fundamental rather than fortuitous they should be found to exist where the spectra are resolved into their polarized components: i.e., the absorption spectrum of each component should consist only of series corresponding to the several fluorescence series of that component and each series of the fluorescence spectrum should have its related series of absorption bands. The rigorous test is difficult because of the incomplete separation of the components in most crystals. That the relation holds however with slight discrepancies, obviously ascribable to the necessarily imperfect conditions as regards polarization, will be seen from figure 4.

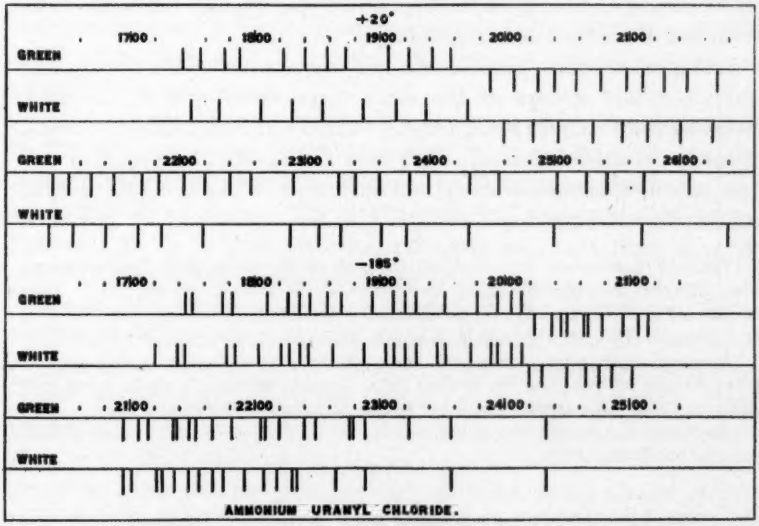

FIG. 5

In this diagram the reversing region, together with two groups of fluorescence bands, to the left, and of absorption bands to the right, is shown for the green and white components of the spectrum. The location of fluorescence bands is indicated by solid vertical lines above the base line, absorption by lines below. Fluorescence is designated by b_g, c_g, d_g, e_g, a_g for the green component and b_w, c_w, d_w, e_w, a_w for the white component. The absorption series related to the above are marked β_g, γ_g, δ_g, ϵ_g, α_g, and β_w, γ_w, δ_w, ϵ_w, α_w respectively.

It will be noted that each series of the green component has its properly related absorption series and that no other absorption bands occur; also that b_g, which is absent or too weak to be observed, is not represented by a related absorption series. The same is true of the

white component excepting that the closely neighboring and feeble γ_w, and β_w are blended into a single intermediate band in each group. The ϵ_w series moreover is slightly shifted towards the violet. The positions given are the averages from five of seven negatives measured.

In figures 3 and 4 only the groups near the reversing region have been indicated. Figure 5 shows all the bands thus far located in the polarized spectra of ammonium uranyl chloride. The diagram is by no means complete however for in the previous studies of the spectrum of this salt, already referred to, at least seven groups of fluorescence bands and eight groups of absorption bands were found. Preparations are in hand for extending the study of the absorption spectra into the ultra violet by means of a quartz spectrograph.

Analogous relations between the two components of the fluorescence and absorption spectra of the other three uranyl double chlorides, $2KCl.\ UO_2Cl + 2HO$; $2RbCl.\ UO_2Cl_2 + 2H_2O$ and $2CsCl.\ UO_2Cl_2 + 2H_2O$, have also been determined. Full data concerning these spectra and that of the ammonium uranyl chloride at $+20°$ and $-185°$ will be published at an early day.

[1] Polarized fluorescence, first described by Grailich (*Krystall-optische Untersuchungen*, Wien, 1858) has since been studied by Maskalyne (*London, Proc. R. Soc.* **28**, 479), V. Lommel (*Ann. Physik., Leipzig*, **8**, 634), E. Wiedemann (*Ibid.*, **9**, 158), Sohncke (*Ibid.*, **58**, 417); G. C. Schmidt (*Ibid.*, **60**, 740), and H. Becquerel, (*Paris, C. R. Acad. Sci.*, **144**, 671).

[2] Becquerel, in the paper already cited, has noted the fact that the fluorescence spectrum is not changed by polarizing the incident light. Sohncke obtained a similar result with calcite and some other crystals but found certain exceptions to the general rule.

[3] See Nichols and Merritt, *Physic. Rev.*, Ser. 1, **27**, 373 (1908); also *Studies in Luminescence*, Carnegie Institution (1912).

THE LINGUISTIC CLASSIFICATION OF POTAWATOMI

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For a considerable period of time it has been commonly supposed that Potawatomi is very closely related to Ojibwa and Ottawa. The statement of William Jones in his *Some principles of Algonquian word-formation*[1] is the most authoritative one on this point. In my *Preliminary report on the linguistic classification of Algonquian tribes*[2] I concluded from my limited first-hand knowledge of Potawatomi that Potawatomi belonged to the Ojibwa group (comprising Ojibwa, Ottawa, Algonkin, Potawatomi; and somewhat removed from them Peoria, etc.) of Central Algonquian dialects, but that Potawatomi differed more from Ojibwa, Ottawa, and Algonkin than these from one another. But

it should be carefully noted that thus far no data have been published to justify these opinions. In the course of the past winter the Bureau of American Ethnology had photostat copies made of three grammatical treatises on Potawatomi which were written by the Rev. Maurice Gailland years ago, but which exist only in manuscript form. In spite of the fact that these treatises are fundamentally arranged on the plan of Latin grammar which is wholly unsuited to the genius of Potawatomi, and that the phonetic system employed is patently inadequate, I have been able to construct nearly complete tables of the verbal pronouns of the independent, subjunctive, and subjunctive of the suppositive modes. From these, as well as from the fact that the negative of the independent mode is formed by means of a *ssi* suffix and that a sibilant is retained before *p*, it follows that the opinions expressed above are completely substantiated.

The personal subjective and objective pronouns which are incorporated in the verb are called 'verbal pronouns' in contradistinction to the independent personal pronouns which in Algonquian languages are used solely for emphasis. The highly specialized character of the verbal pronouns in the Central Algonquian dialects has made them thus far the most satisfactory classificatory criteria.

The decisive verbal pronouns showing the close relationship of Potawatomi to Ojibwa, Ottawa, and Algonkin are:

Independent mode: I—you, I—him, I—them (animate), we (exclusive)—thee, we (exclusive)—you, we (exclusive)—it, we (exclusive)—them (inanimate), we (inclusive)—it, we (inclusive)—them (inanimate), thou—him, thou—them (animate), ye (intransitive), ye—me, ye—him, ye—them (animate), he—you, he—him, they (animate)—us (exclusive), they (animate)—us (inclusive), they (animate)—you, they (animate)—him, they (animate)—them (animate).

Subjunctive mode: I—them (animate), we (exclusive)—thee, we (exclusive)—you, thou—them (animate), they (animate)—thee, they (animate)—him, they (animate)—them (animate), they (animate)—it, they (animate)—them (inanimate).

Subjunctive of dubitative mode: They (animate, intransitive).

The verbal pronouns showing the divergence of Potawatomi are:

Independent mode: we (exclusive)—him, we (inclusive) them (animate), we (inclusive)—him, we (inclusive)—them (animate), ye—it.

Subjunctive mode: I—you, we (inclusive)—him, we (inclusive)—them (animate), he—us (exclusive).

Subjunctive of the suppositive mode (corresponding to the Fox subjunctive of the interrogative mode): ten verbal pronouns.

By consulting the table of verbal pronouns on the folder at the end of my *Preliminary report*, the reader will at once see the importance of the pronouns mentioned above, in determining the linguistic classification of Potawatomi. Complete tables for Potawatomi will be published in the *American Anthropologist*.

[1] *Amer. Anthrop., New York*, N. S., 6, 369–411.
[2] *Smithsonian Inst., Rep. Bur. Amer. Eth.*, 28, 221-290b.

THE LIGHT CURVE OF XX CYGNI AS A CONTRIBUTION TO THE STUDY OF CEPHEID VARIATION

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Presented to the Academy, June 29, 1915

Probably as many as 90% of the 4500 stars whose light intensities are known to be variable may be placed in three well-defined classes. (1) The eclipsing variables are of various colors (or spectral types); their periods range from a few hours to a year or more in length, and their variation may be from less than one-tenth to several stellar magnitudes. (2) The long period variables are generally reddish stars; the cycle of their light-changes occupies from one hundred to six or seven hundred days, and the amount of their light variation is usually several magnitudes. (3) The largest class, however, since it includes the great number of variables in globular clusters and in other special regions of the sky, is that known as short period variables or Cepheid variables. Practically all types of spectra are represented, though types A and F predominate so far as now known. The periods in general are much less than 50 days, and for a very large subdivision (the cluster type variables) average about 12 hours. The variation, which is practically continuous, is nearly always of the order of one magnitude in range, and is characterized generally by a more rapid increase than decrease of brightness.

The cause and characteristics of eclipse variation are definitely known. But this can not be said of the other two main types of variable stars. Though many suggestions have been made, the true cause of long period variation remains more or less obscure. The interpretation of Cepheid variables has been much debated and much evidence in favor of various theories has been collected, but no explanation has as yet received general acceptance. The behavior of the spectral lines of Cepheids has led to the widely adopted assumption that they are spectroscopic binaries and that the light variations are in some way related to orbital

motion. But the many serious obstacles in the way of the double star hypothesis have been pointed out in a previous paper,[1] and the suggestion made that the observed facts may accord much better with the assumption that the light and velocity variations are both due to disturbances on a large scale of the radiating surfaces of single stellar bodies. Such periodic disturbances, which have in sun-spots a not very distant analogue both as to spectral displacements and light variations, would have as an underlying cause the long-enduring free oscillations that may be set up in a gaseous mass in a variety of ways, such as the collision with or close approach to another body, the perturbations by an obscure companion, or the adjustment of mass or temperature dissymmetries.

Among the arguments against the double star interpretation of Cepheids is their property of definitely changing color (therefore, of changing surface temperature) with the variation in the brightness of the star, thus indicating that the phenomenon is peculiar to the radiating surface and not merely geometrical as with the eclipsing double stars. Another point is the lack of regularity in the time of the principal phases and in the nature of the light variations. It is in connection with these two arguments that the present investigation hopes to contribute to the explanation of the Cepheid variables. In particular the present discussion of the light variations of the star XX Cygni challenges the frequently iterated assertion that the short period variable, because of its supposedly great precision and regularity of performance, is a veriable timepiece. In fact the suggestion is on record that the unit of time might possibly be based on the periodic return to maximum brightness of certain short period variables rather than on the rotation of the earth. While in most cases so far as now known the *mean* periods of such variables have, to be sure, but little or no secular or periodic variation, the study of XX Cygni, in agreement with preliminary results for similar variables, has shown definitely that the light curves change rapidly and, it may be, erratically both in time of maximum and in character of variation. The details of this work will appear in the *Astrophysical Journal*;[2] some of the chief results are outlined below.

In a monograph on the light variations of XX Cygni, Kron gives for the visual range 0.76 mag., based on the observations of seven observers, and for the photographic range, from observations by Parkhurst and Jordan, 0.63 mag.[3] This result is unusual in that the visual variation is the larger; every other variable star for which we have such information has a greater range photographically than visually. The difference between the ranges is a direct measure of the change in

color. That the star with the shortest known perod (3.2 hours) should show an abnormal color variation suggests that for some unknown reason it may be a real exception. To determine whether or not this is the case, observations were undertaken at Mount Wilson with the 60-inch reflector. Four series of plates were obtained (in all more than 300 exposures); each of the first two covers a complete period, and as Seed 27 plates were alternated with isochromatic plates (used with a yellow color filter), each series yields for the study of color phenomena complete, simultaneously determined photographic and photovisual light curves.

The photovisual curves thus obtained show no great deviation from Kron's mean visual curve. The ranges are 0.72 mag. and 0.85 mag., as compared with Kron's mean of 0.76 mag. The curves for the two nights differ in certain particulars from each other, and as both are smooth and well determined, the observed differences are probably real. This belief is supported by an investigation of the visual observations published by Kron, which show a much greater diversity in the shape of the nightly curves than can possibly be accounted for by errors of observation. There are, in general, two types of maximum; one is narrow and pointed, the other is broad, round-topped, and somewhat lower. The accompanying figure gives examples taken from the photometric observations by Guthnick at Berlin.

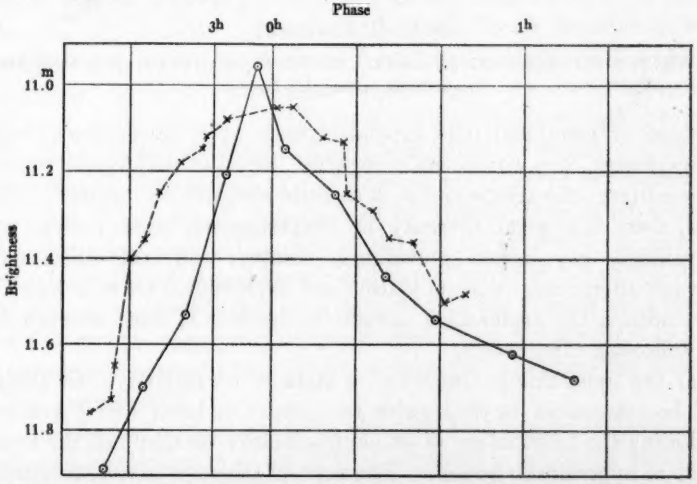
The Mount Wilson photographic curves differ from each other in the same way as the photovisual; for the same maximum the photographic and photovisual curves are very similar in form but differ from those of the other night.

A comparison of the Mount Wilson curves yields the following results: The photographic maximum occurs simultaneously with the photovisual within the errors of observation. The magnitudes at minimum phase agree for the two nights, being photovisually 12.17 and 12.17, and photographically 12.50 and 12.49, respectively. This gives a color index at minimum of +0.32, corresponding to spectral type A8 if the usual relation between color index and spectrum is adopted for this particular case. In the magnitudes at maximum, however, there is no such agreement. Photovisually they are 11.45 and 11.32, photographically 11.53 and 11.74, so that the color index at maximum varies from +0.08 to +0.42.

It is thus seen that so far as our data are decisive the photographic range is not constant. On one night it is 0.97 mag., on the other 0.75 mag. The smoothness and definiteness of the curves makes it impossible to attribute this large difference to observational errors entirely.

The constance of light at minimum and the evident variation at maximum leads to the conclusion that the minimum phase is the normal condition of the star, and suggests that the maxima represent fairly equally spaced but otherwise irregular and varied light disturbances.

As to whether the photographic or the visual range is the greater (the ratio of the two may not be constant), the data seem to indicate that the former is at least equal to the latter, if it does not, in fact, considerably exceed it, so that XX Cygni probably is not materially different in the matter of color change from other variables of the cluster or Cepheid type.



GUTHNICK'S VISUAL LIGHT CURVES OF XX CYGNI FOR SEPTEMBER 22, AND OCTOBER 5 (BROKEN LINE), 1908, SHOWING TWO EXTREME TYPES OF MAXIMUM.

Summary. (1) The cause of light variation has not been ascertained definitely for the variable stars known as Cepheids or short period variables, but in preference to the hypothesis that they are double stars an alternative explanation is proposed that seems to harmonize much better with observed data. (2) The new hypothesis would ascribe the periodic light and spectrum variations to vibrations in isolated stellar masses. (3) The present study of the variable XX Cygni is based upon 300 Mount Wilson photographic observations in which the variation for light of different wave-lengths is considered, and upon the analysis of 2700 visual measures by European astronomers. (4) It contributes to the solution of the problem by showing that a supposed exception to the normal color phenomena of Cepheids probably does not exist. (5) Of more importance to the Cepheid interpretation it

shows that the periodic occurrence of maximum, far from being clock-like in its precision, is distinctly irregular in phase and in other details.

[1] *Mt. Wilson Contr.* No. 92; *Astrophys. J.*, **40**, 448 (1914).
[2] A Study of the Light Curve of XX Cygni, *Mt. Wilson Contr.* No. 104; *Astrophys. J., in press* (1915).
[3] *Potsdam, Publ. Astrophysik. Obs.*, **22**, Part III (1912).

THE FEEBLY INHIBITED. III. INHERITANCE OF TEMPERAMENT; WITH SPECIAL REFERENCE TO TWINS AND SUICIDES

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Presented to the Academy, June 29, 1915

Mood or emotional tone depends largely upon temperament; and temperament, it is commonly recognized, has a constitutional basis—is hereditary; the temperament is as little alterable as stature. However, there is a great diversity in temperaments; some persons are prevailingly gay; others prevailingly somber; and still others pass through alternating cycles of elation and depression. Of so complex a phenomenon the explanation cannot be simple. It must account for the following three states:

(*a*) the hyperkinetic state—i.e., a state of overactivity. Of this it will be convenient to distinguish two grades, a lesser and a greater. Following the terminology of an old psychology we may call the lesser grade of hyperkinesis *nervous*. The nervous person is active, energetic, irritable, excitable, ambitious, given to planning, optimistic; usually talkative and jolly. The greater grade of hyperkinesis is the choleric. A choleric person is apt to start a new line of work before he completes the old, brags, is usually hilarious, hyperotic, often profane, liable to fits of anger, brutal, destructive, assaultive, and even impulsively homicidal and suicidal.

(*b*) The opposite state is the hypokinetic or depressed. Here too we may distinguish two grades. The phlegmatic person is quiet, serious, conservative, pessimistic. The melancholic person is unresponsive (often mute), lachrymose, given to worry, weak and incapable, feels life a burden, often longs for death as a relief.

(*c*) The normal mood is shown in that the possessor is cheerful without being boisterous, calm, well balanced, and *en rapport* socially; he works and plays moderately, laughs quietly, does not weep easily, feels little drive, and is always responsive and coöperative. This

state may be due either to the absence of excitation or the absence of depression.

Now each of the three states of hyperkinesis and its absence may be combined with any of the three states of hypokinesis and its absence, making nine combination-states. And since there are two parents in any mating the number of different matings possible will be the sum of the series $9 + 8 + \ldots + 1$, or 45.

To account for this variety of temperaments, hyperkinetic, hypokinetic, normal and mixed, various hypotheses were tried and finally the following selected as most fitted to explain the facts on an hereditary basis. *There is in the germ plasm a factor E which induces the more or less periodic occurrence of an excited condition (or an exceptionally strong reactibility to exciting presentations) and its absence, e, which results in a calmness. There is also the factor C which makes for normal cheerfulness of mood, and its absence, c, which permits a more or less periodic depression. Moreover, the factors behave as though in different chromosomes, so that they are inherited independently of each other and may occur in any combination.*

What the nature of these factors is, whether they affect primarily the development of certain parts of the nervous system or the secretions of certain glands, is not known and is not involved in the hypothesis. It is even conceivable that each state may be due to more than the pair of factors here suggested; but if the hypothesis fits the facts it would indicate that in the factors *E* and *C* we have the predominating influences that control mood.

For the test of the hypothesis 89 carefully described family histories were available, and these afforded 147 matings in which the mated pair, their parents (usually), and certain of their offspring were sufficiently described for the purposes of the test.

The test of the hypothesis is found in a comparison of the expected and actual distribution of temperaments in the children of each sort of mating. Of the 45 possible matings 29 were realized. The relations of the sums of the observed to the sums of the expected distributions among the nine classes of temperaments is shown in the following table.

choleric cheerful	*choleric phlegmatic*	*choleric melancholic*	*nervous cheerful*	*nervous phlegmatic*	*nervous melancholic*	*calm cheerful*	*calm phlegmatic*	*calm melancholic*
36	25	30	128	149	63	77	79	46
41–6/8	46–7/8	19–5/8	98–4/8	154–6/8	72–6/8	51–4/8	97–3/8	49–7/8

There are several reasons for not expecting a very close relation between observed and expected; but the chief one is that there are sometimes several 'expected' results; and in these cases the most varied result was taken as the 'expected.' Nevertheless, the relation between

the two series is fairly close—the greatest difference being an unexpected excess of choleric-melancholics and corresponding deficiency of choleric-phlegmatics; also an excess of calm-cheerful and deficiency of calm-phlegmatic. These discrepancies imply a rather slight error in the classification of the observed cases.

We conclude, then, from our own data that the hypothesis is confirmed. In addition, an examination of the literature reveals clear evidence that a difference in the inheritance of extreme hyperkinesis (a dominant) and extreme hypokinesis (a recessive) has unconsciously been observed. And the differences in the conclusions of Rosanoff and Orr[1] and Rüdin[2] concerning the inheritance of manic-depressive insanity—one regarding it as recessive and the other regarding it as *sometimes* dominant—are easily explained on the ground of its complex hereditary nature.

The hereditary nature of temperament is demonstrated by the facts of the personal history of identical twins, as given by Galton.[3]

There is reason for thinking that the different zygotic combinations of temperamental factors occur with different frequencies. A comparison of the relative number of matings in equally frequent kinds of zygotic combinations shows that some are much more common than expected, others much less. The mated pair rarely have the same zygotic temperamental formula. Two choleric or 'nervous' persons frequently do not marry each other. Two melancholics rarely intermarry. There is, in marriage, a selection against similar temperaments, i.e., a preference for those of more or less markedly dissimilar temperaments.

An analysis of the temperaments of suicides shows that they fall into the two types of the hyperkinetic (or impulsion) suicides and the hypokinetic (or depressed) suicides. Also, a family tendency to suicide by the same method is evident; but it is uncertain whether this is due chiefly to subconscious suggestion or to an hereditary bias.

This study throws light upon the nature of the 'functional insanities' and lends strong support to the view that they are syndromes whose elements are separately inheritable.

Finally the study throws light upon the 'springs of conduct.' Just what we shall, in any situation, do is determined by numerous factors, but the general nature of our reactions, whether violent or repressed—this is determined by the hereditary nature of our temperaments. The romantic and the classic type of reacting, the hyperkinetic and the hypokinetic, the radical and the conservative, the feebly inhibited and the strongly inhibited constitute a dualism that runs through our whole population.

It is expected that the complete paper will appear as a publication of the Carnegie Institution of Washington.

[1] Rosanoff, A. J., and Florence Orr, A study of insanity in the light of the Mendelian theory, *Amer. J. Insanity*, 68, 221–261 (1911), also *Eugenics Record Office Bull.*, No. 5.

[2] Rüdin, E., Einige Wege und Ziele der Familienforschung, mit Rucksicht auf die Psychiatrie, *Zs. ges. Neurol. Psychiat.*, 7 (Hft. 5), 487–585 (1911).

[3] Galton, F., History of Twins, in *Inquiries into human faculty and its development* (1883), reprinted in 'Everyman's Library,' London, Dent, N. Y., Dutton.

SECOND TYPE STARS OF LOW MEAN DENSITY

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The rôle played by stellar densities in the theories of the order of stellar evolution is easily recognized. If the development of a star uniformly follows the spectral sequence B, A, F, G, K, M, that is, if in contracting through the agency of its own gravitation its light changes in color steadily from bluish white to red, then the mean density of the B and A type stars must naturally be less than that of the redder stars. But, on the other hand, if the earlier as well as the later stages in a star's evolution are characterized by the second and third type spectra (F, G, K, M), the density of the redder stars should have both small values and large values relative to the bluer stars.

There are two principal lines of observational evidence that indicate the existence of two orders of densities for second type stars:

1. The recognized existence of enormous differences in the absolute brightness of red or yellow stars of essentially identical spectra leads immediately to the conclusion that the volumes also differ greatly. Now since stellar masses so far as known have not very greatly differing values, we are lead to believe that the disparity in size must be accounted for by large differences in mean density. For instance, the density of our sun, which on this interpretation must be considered a body relatively advanced in age, may be from a thousand to a million times greater than that of a star of great intrinsic luminosity, such as Alpha Aurigae, which has a similar spectrum and presumably therefore a comparable suface temperature. An objection to this argument in favor of very low densities for some of the redder stars may be that our information relative to stellar masses is insufficient to prove that the large volumes are not to be attributed to large masses with normal densities.

2. The densities of eclipsing binaries, however, can be determined directly, independently of the unknown masses; and recent studies of

this class of variable stars have given results for different spectral types with considerable certainty. The average density of first type stars (B, A) is about 0.15 on the solar standard, while none exceeds the sun in density and none is less than one-hundredth as dense. Also a number of second type stars are found to be much denser than the average first type star, even exceeding the sun in rare cases. This condition fits in with either theory of the order of stellar evolution. Hence for the purpose of differentiating between the rival hypotheses it is of importance to know how definitely we can say that there are also stellar densities much lower than any known for A and B type stars. The present communication outlines the data now in hand relative to low-density eclipsing stars of the second spectral class.

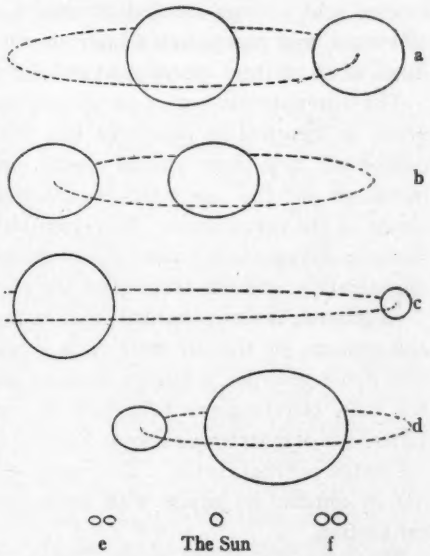
The values of the densities already derived for the stars considered in this note refer to the individual components of each system, and therefore, since for such purpose knowledge of the relative dimensions and masses is necessary, the results are involved with rather complicated computations of the orbital elements. It is possible, however, to derive upper and lower limits of the mean density of each system without the necessity of any data relative to the stars or their light curves other than the period of revolution and the duration of eclipse, which are the two quantities most commonly and most easily obtained for eclipsing variables. In the more detailed discussion of this subject the following simple relations are obtained, which give in a circular orbit the mean density, d_0, in terms of the sun:

$$d_0 \leqq \frac{0.054}{P^2 \sin^3 \frac{2\pi t}{P}}, \qquad d_0 > \frac{0.216}{P^2 \left(1 + 3 \sin^2 \frac{2\pi t}{P}\right)^{3/2}}.$$

The quantities P and t are the period of revolution and the semi-duration of eclipse. In very exceptional cases the second relation may give too high a lower limit, but never a value as much as twice too large.

The above formulae are especially suitable for the easy computation of the order of the density in systems of long period, which are usually also the cases where very low densities are found. With their aid results for five low density binaries of the second spectral type were readily determined and the values found to correspond closely to those computed rigorously from the orbital elements. The light curves on which the results are based are from various sources, and, while not of the first order of accuracy, are probably sufficient in all cases to leave no doubt that the low densities really exist. Three independent classifications of the spectra were made at the Harvard College Observatory.

In the accompanying figure is given a comparative diagrammatic representation of some of these giant eclipsing double stars and also of the two systems of highest known density. The key to the diagram gives the data relative to densities and spectra. Making the reasonable assumption that the mass of the sun is sensibly equivalent to the mass of one component of any of the double stars, then the sun can be drawn to scale, as at the bottom of the diagram. If its mass is but one eighth that of a single component, its diameter in the diagram should be one-half as great; if eight times as massive as assumed, its diameter should be doubled. The true dimensions of the stars relative to the sun are probably well within these limits. It is to be noticed that the components of five of the six binaries are elongated in the line of centers.



Eclipsing binaries of low and high density. The mass of each component is assumed equal to that of the sun.

a. SX Cassiopeiae; period $36^{d}.572$; spectrum G3; density 0.0003. Reduced to one-half.

b. RX Cassiopeiae; period $32^{d}.316$; spectrum Ko; density 0.0005. Reduced to one-half.

c. RZ Ophiuchi; period $261^{d}.9$; spectrum F8; density 0.001. Reduced to one-fourth.

d. W Crucis; period $198^{d}.5$; spectrum Gp; density 0.000002. Reduced to one-eighth.

e. W Ursae Majoris; period $0^{d}.334$; spectrum G; density 1.84.

f. U Pegasi; period $0^{d}.375$; spectrum F?; density 0.75.

Summary. Because of its bearing on the question of the order of stellar evolution, the density of stars of the second spectral type has been discussed from the standpoint of the dependability of the observational and theoretical work that is the basis of the derivation of occasional extremely low values. The evidence discriminating between the two principal theories seems very definite, but further observational work would materially strengthen the case. The light curves of the variable stars involved, the relevant tabular data, and the derivation of certain density relations will appear in an early number of the *Astrophysical Journal*.

ON THE PATHOLOGICAL ACTION OF ARSENICALS UPON THE ADRENALS

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Presented to the Academy, July 3, 1915

The fact that arsenicals of diverse chemical constitution exert a pronounced pathological action upon the adrenals has not been generally recognized. The importance of this adrenotropic action was first impressed upon us while carrying out the routine biological tests of compounds of arsenic in our chemotherapeutic studies. Observations on more than sixty compounds including such substances as arsenious acid, arsenic acid, sodium cacodylate, atoxyl, arsacetin, arsenophenylglycine, salvarsan, and neosalvarsan have shown that, without exception, toxic doses of all of these arsenicals produce definite lesions of the adrenals.

The adrenotropic action of all compounds of arsenic is not equally great or identical in character but the lesions produced by a given compound in a given animal species are quite constant and in some instances are the dominant pathological manifestations of the toxic action of the compounds. The essential features of this action concern vascular changes in the adrenal, alterations in the lipoid content, cellular degeneration, and the effect upon the chromaffin.

In general, arsenical intoxication in the guinea pig produces an acute enlargement of the adrenals with some congestion and hemorrhage. The lipoid granules, normally demonstrable in the outer half of the cortex with Herxheimer's Scharlach R., first appear as larger droplets. Later, the demonstrable lipoids increase in amount and are spread over the entire adrenal cortex. This stage of lipoid increase is succeeded by one of diminution which with some compounds progresses almost to exhaustion.

The cells of both the cortex and the medulla show a variety of degenerative changes and even necrosis; colloid degeneration of the medulla is particularly striking. While the cortex is slightly infiltrated with leucocytes the accumulation of both leucocytes and polyblasts in the medulla is especially marked with such substances as arsenophenylglycine.

Regeneration of cortical cells by mitosis is very active after forty-eight hours and mitotic figures are occasionally seen in the cells of the medulla.

The effect of arsenicals upon the chromaffin is of especial interest. Some compounds seem to exercise but slight influences upon the chro-

maffin content of the adrenals while others, such as sodium cacodylate, salvarsan, and neosalvarsan, cause a rapid and marked decrease in this substance, as judged by the color of the medullary cells after fixation in Müller's fluid.

From these observations it appears that the adrenotropic action of arsenicals is one of the most constant and important features of arsenical intoxication and we suggest that therapeutic doses of some arsenicals may be found to produce definite stimulation of the adrenal glands.

VARIATIONS IN THE CHARACTER AND DISTRIBUTION OF THE RENAL LESIONS PRODUCED BY COMPOUNDS OF ARSENIC

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The chemical agents employed in the production of experimental nephritis are usually divided into two classes, those producing tubular lesions and those producing vascular lesions, to the latter group of which the compounds of arsenic have been assigned. From a study of the renal lesions produced by a large number of arsenicals, however, we have been led to question the validity of such a classification and to view the pathogenic action of such substances from the standpoint of their chemical constitution as well as their arsenic content.

The classical hemorrhagic kidney of arsenious acid is by no means constant for all compounds of arsenic. Grossly, the kidneys of dogs given lethal doses of such substances as arsenious acid, salvarsan, neosalvarsan, galyl, arsenophenylglycine, atoxyl, and arsacetin are separable into two extreme groups, the red and the pale kidneys, with transitional types in which the predominating changes ally them more closely with the one group or the other. In the group of red kidneys, congestion and hemorrhage are the dominant features of the arsenical action, while in the pale kidneys, the dominant lesion is tubular.

Upon closer analysis of the gross and microscopic changes we can make a further differentiation of the action of compounds that produce kidneys of the one or the other of these types. For example, arsenious acid, salvarsan, neosalvarsan, and galyl all produce red kidneys, but the congestion and hemorrhage produced by arsenious acid is diffuse in character with but slight tubular necrosis, while the vascular injury of salvarsan, neosalvarsan, and galyl is more pronounced in the cortex and the boundary zone and is accompanied by much more marked

tubular necrosis and interstitial edema. Again, atoxyl causes extensive hemorrhage extending from the boundary zone through the medulla and only slightly into the cortex, the outer rim of which remains pale; there is marked degeneration and necrosis of tubular epithelium. These atoxyl kidneys which are outwardly pale combine in a peculiar form the essential features of both the red and the pale kidneys. In like manner, arsenophenylglycine produces kidney lesions of a combined type and while tubular degeneration and necrosis are dominant there is usually some congestion and hemorrhage in the boundary zone and medulla—more rarely in the cortex.

The other extreme in the action of arsenicals upon the kidney is exemplified by arsacetin which produces a typically pale kidney. While congestion and hemorrhage are still apparent to a minor degree in the boundary zone of these kidneys the vascular injury is so completely overshadowed by the injury to the tubular epithelium as to leave no doubt as to the dominance of tubular injury. Further, the prompt and vigorous regeneration of the tubular epithelium indicates that the extensive necrosis produced by arsacetin can not be regarded as a secondary anemic phenomenon.

It is certain, therefore, that all arsenicals do not produce renal lesions that are identical either in character or distribution but that this group of substances includes agents producing a so-called tubular nephritis as well as those producing a vascular nephritis, and that these wide differences in the pathogenic action of different compounds of arsenic are explainable only upon the basis of their chemical constitution.

SEVEN POINTS ON A TWISTED CUBIC CURVE

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Presented to the Academy, August 2, 1915

Six points in space, barring special situations, determine a twisted cubic curve. From any seventh point of the curve those six are projected by six generators of a quadric cone. For any seven points of a cubic curve there is accordingly a symmetric set of seven cones; and it is well known that seven points giving rise to two such cones are on a cubic curve, and so give rise to five more cones. This is the only current theorem on seven points of a twisted cubic. Concerning eight points there is the elegant theorem of von Staudt, that if two tetraedrons have eight points of a twisted cubic for vertices, their eight faces osculate a second cubic curve. I propose to demonstrate a theorem whose formulation resembles the latter, while like the former it relates to the

fewest possible points, seven. It is this: *If seven points on a twisted cubic be joined, two and two, by twenty-one lines, then any seven planes that contain these twenty-one lines will osculate a second cubic curve.*

The proof to be given is analytic, consisting in a repeated application of Pascal's theorem concerning a hexagon inscribed in a conic, adapted to the six projecting lines from a seventh point by means of Clebsch's so called *Ränderungs-princip*.

Pascal's theorem may be expressed in notation readily understood, for six points a, b, c, d, e, f of a conic (coordinates (a_1, a_2, a_3) etc.) by the vanishing of the determinant

$$\begin{vmatrix} ((ab),(de))_1 & ((bc),(ef))_1 & ((cd),(fa))_1 \\ ((ab),(de))_2 & \cdots\cdots & \cdots\cdots \\ ((ab),(de))_3 & \cdots\cdots & \cdots\cdots \end{vmatrix}$$

or any one of the 59 equivalent forms. This is reducible to the equation, in three-rowed determinants of point-coordinates,

$$(abe)\,(cde)\,(adf)\,(bcf) = (abf)\,(cdf)\,(ade)\,(bce). \qquad (1)$$

(See *Encyklopädie der mathematischen Wissenschaften*, 3_2, §7.)

Clebsch's *Ränderungs-princip* applied to this equation gives the condition that six points in space (a_1, a_2, a_3, a_4), etc., or briefly a, b, c, d, e, f, shall be projected from a seventh point $(g_1, g_2, g_3\ g_4)$ by six generators of a quadric cone, viz.,

$$(abeg)\,(cdeg)\,(adfg)\,(bcfg) = (abfg)\,(cdfg)\,(adeg)\,(bceg). \qquad (2)$$

If the seven points are on a twisted cubic, this relation may be written in $7 \cdot 15 \cdot 3$ different ways, according to the choice of the seventh point, g, the two points e and f, and the two ways of forming pairs from the remaining four points a, b, c, d. Let equation (2) denote all of this type.

Seven planes, 1, 2, 3, 4, 5, 6, 7 may be determined in 30 different ways so as to contain the 21 lines that join pairs of the points a, b, c, d, e, f, g. Any one such choice constitutes a *triad system* on those seven letters, since its seven sets of three points in a plane must contain every pair of points once and only once. Fix one set thus, planes, 1, 2, 3, 4, 5, 6, 7 containing the points *ade, afg, bdf, beg, cdg, cef, abc*, respectively. These seven planes, in order to osculate a twisted cubic, must satisfy conditions on planes precisely dual to the 315 conditions of type (2) on points. On account of their similarity we need to verify only one. This is a slightly tedious reckoning, which we give *in extenso*.

The question shall be: Does the plane 7 intersect the six planes 1, 2, 3, 4, 5, 6 in six tangents to a conic? Is the following condition verified?

$$(1257)\,(3457)\,(1467)\,(2367) = (1267)\,(3467)\,(1457)\,(2357). \qquad (3)$$

Now it is necessary to express the coordinates of each plane as three-rowed determinants in the coordinates of the three points which it contains, to simplify, and compare the left hand product of four determinants in (3) with that on the right, and to show that equations (2) above will suffice to prove the equality so reduced. On the left:

$$\begin{aligned}(1257) &\equiv (\,(ade)_1\,(afg)_2\,(cdg)_3\,(abc)_4)\\ &\equiv (adeb)\,(afgc)\,(cdga) - (adec)\,(afgb)\,(cdga)\\ &\equiv (cdga)\,\{\,(adeb)\,(afgc) - (adec)\,(afgb)\,\},\\ (3457) &\equiv (cdgb)\,\{\,(bdfc)\,(bega) - (bdfa)\,(begc)\,\},\\ (1467) &\equiv (adeb)\,(begc)\,(cefa) + (adec)\,(bega)\,(cefb),\\ (2367) &\equiv (afgb)\,(bdfc)\,(cefa) + (afgc)\,(bdfa)\,(cefb).\end{aligned}$$

On the right:

$$\begin{aligned}(1267) &\equiv (cefa)\,\{\,(adeb)\,(afgc) - (adec)\,(afgb)\,\},\\ (3467) &\equiv (cefb)\,\{\,(bdfc)\,(bega) - (bdfa)\,(begc)\,\},\\ (2357) &\equiv (afgb)\,(bdfc)\,(cdga) + (afgc)\,(bdfa)\,(cdgb),\\ (1457) &\equiv (adeb)\,(begc)\,(cdga) + (adec)\,(begc)\,(cdgb).\end{aligned}$$

Notice that on the left two factors in the {} are identical with two in {} on the right. Exclude these, when multiplication gives in each member four terms. Two terms on the left are identical with terms on the right. Remove these, and transpose so as to exhibit the factor

$$(cefa)\,(cdgb) - (cefb)\,(cdga)$$

on each side. Neglect this, and our question is reduced to the following. Is it true that

$$\begin{aligned}(abfg)\,(abde)\cdot(bcdf)\,(bceg)\ \cdot(acdg)\,(acef)\\ = (abdf)\,(abeg)\cdot(bcdg)\,(bcef)\cdot(acde)\,(acfg)\ ?\end{aligned}\qquad (4)$$

Two applications of relations like (2) prove the truth of this, for example we use first b, then c, as seventh point like the g of (2);

$$(abfg)\,(abde)\,(bcdf)\,(bceg) = (abdf)\,(abeg)\,(bcfg)\,(bcde),$$

and

$$(bcfg)\,(bcde)\,(acdg)\,(acef) = (bcdg)\,(bcef)\,(acfg)\,(acde).$$

These two Pascalian relations upon the seven points are part of the hypothesis, hence the Pascalian equation (3) upon the seven planes is verified and the theorem is proved.

It is interesting to restate this theorem somewhat more fully. The converse is true by duality, from seven planes to any seven points that lie on all 21 of their lines of intersection. Consequently

If seven planes intersect three and three in seven points of a twisted cubic, each of the 29 other sets of seven points that contain all the 21 lines of intersection of those planes is upon another twisted cubic curve.